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AFRL-SR-AR-TR-04-

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1. REPORT DATE (DD-MM-YYYY)	2. REPORT TYPE	3. DATES COVERED (From - To)
	Final Report	Dec 1, 99 - Nov 30, 03

4. TITLE AND SUBTITLE	5a. CONTRACT NUMBER
Dynamic Fracture in Brittle Materials	
	5b. GRANT NUMBER
	F49620-00-1-0003
	5c. PROGRAM ELEMENT NUMBER
6. AUTHOR(S)	5d. PROJECT NUMBER
Dr. Jay R. Walton	
	5e. TASK NUMBER
	5f. WORK UNIT NUMBER

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)	8. PERFORMING ORGANIZATION REPORT NUMBER
Texas A&M University College Station, TX 77843	

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)	10. SPONSOR/MONITOR'S ACRONYM(S)
Department of the Air Force Air Force Office of Scientific Research	
4015 Wilson Blvd. Arlington, VA 22203-1954	
	11. SPONSOR/MONITOR'S REPORT NUMBER(S)

## 12. DISTRIBUTION/AVAILABILITY STATEMENT

Distribution Statement A: Approved for public release. Distribution unlimited

20040225 168

13. SUPPLEMENTARY NOTES
DODAAC CODE: 1BZC2
Program Manager: Dr. Arje Nachman

## 14. ABSTRACT

Substantial progress has made in three of the proposed problem areas as described below. Moreover, the P.I.'s research evolved to other areas not in the proposal but relevant to the same material application areas. These contributions are also discussed below:  
Dynamic Crack Growth in Functionally Graded Material:

The P.I. and his Ph.D. student, David Zeigler (degree conferred August 2001), considered the problem of dynamic crack growth along an interface in a functionally graded composite material. In such a material, the constitutive properties vary smoothly away from a bi-material interface. Such composite material systems are of increasing importance in technological applications since they offer the prospect of creating laminated composite materials that are not susceptible to the debonding failure occurring at traditional bi-material interfaces due to the mismatch in material properties.

## 15. SUBJECT TERMS

16. SECURITY CLASSIFICATION OF:	17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
a. REPORT	b. ABSTRACT	c. THIS PAGE	Dr. Jay R. Walton
			19b. TELEPHONE NUMBER (Include area code)

FINAL TECHNICAL REPORT

December 2003

**DYNAMIC FRACTURE IN BRITTLE MATERIALS**

AFOSR GRANT NO. F49620-00-1-0003

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**Objectives.**

The primary objective of this project is the formulation and analysis of canonical boundary value problems modeling dynamic fracture in brittle materials. The proposed objectives were to address the following problem areas.

1. **Unsteady Mode I Crack Growth in Elastic Material:** The P.I. proposed to generalize his previous analysis of the dynamic, transient growth of an antiplane shear crack in an infinite linear elastic body to the more difficult, but physically more important, Mode I (opening mode) fracture setting. One of the goals of this line of research is to investigate the manner in which a macrocrack coalesces with a co-planar microcrack since there is considerable experimental evidence that this process plays a critical role in dynamic crack growth in brittle materials.
2. **Dynamic Crack Growth in Functionally Graded Elastic Bodies:** Functionally graded materials are becoming increasingly important in technological applications. However, few analyses of cracks in functionally graded material appearing in the literature have considered dynamic crack growth with realistic material properties. The P.I. proposed to generalize his work on dynamic crack growth in homogeneous elastic bodies to functionally graded material. Specifically, the P.I. proposed to consider the unsteady growth of a straight crack in a functionally graded material whose elastic properties vary both in the direction transverse to the crack plane and in the direction of crack propagation. The P.I. proposed to pursue an asymptotic approach as a means of gaining insight into the boundary value problems corresponding to realistic material models. The P.I. proposed then use these asymptotic solutions to address the question of designing functionally graded materials with optimal fracture properties.
3. **Temperature Effects in Dynamic Crack Growth:** It has been observed experimentally in many materials that significant temperature increases occur

within a thin neighborhood surrounding a dynamically propagating crack tip. Moreover the P.I. and his collaborator Francesco Costanzo have shown within the context of the dynamic steady propagation of a semi-infinite anti-plane shear crack in an infinite linear elastic body that many of the experimentally observed dynamic fracture phenomena that seem to run counter to classical linear elastic fracture mechanics predictions follow naturally from modeling the crack tip region by a temperature sensitive cohesive zone. The P.I. proposed to extend this work to the opening mode (mode I) fracture setting. Two approaches were to be pursued. The first is to generalize the combined analytical/numerical methods used previously on the mode III problem to mode I context, and the second is to assist Dr. Costanzo's project to develop a direct numerical simulation scheme based upon a discontinuous Galerkin methodology that he has been pursuing to treat general elastodynamic problems with moving singularities.

4. **Dynamic Crack Growth in Viscoelastic Bodies:** Fiber reinforced composites have myriad uses as structural materials in Air Force applications as well as everyday life. Understanding the fracture behavior of such materials is of paramount importance. Many fiber reinforced composites utilize polymeric binders which exhibit considerable hysteresis under cyclic loading. The P.I. proposed to study the dynamic steady-state problem of a semi-infinite crack propagating in an infinite anisotropic viscoelastic body under generalized plane strain deformations with fully mixed mode loading (combined modes I, II and III). The viscoelasticity is intended to model a polymeric binder matrix while the anisotropy models fiber orientation.

#### **Status of Effort and Accomplishments.**

Substantial progress has made in three of the proposed problem areas as described below. Moreover, the P.I.'s research evolved to other areas not in the proposal but relevant to the same material application areas. These contributions are also discussed below.

**Dynamic Crack Growth in Functionally Graded Material.** The P.I. and his Ph.D. student, David Zeigler (degree conferred August 2001), considered the problem of dynamic crack growth along an interface in a functionally graded composite material. In such a material, the constitutive properties vary smoothly away from a bi-material interface. Such composite material systems are of increasing importance in technological applications since they offer the prospect of creating laminated composite materials that are not susceptible to the debonding failure occurring at traditional bi-material interfaces due to the mismatch in material properties. They studied the problem of a semi-infinite, anti-plane shear crack propagating dynamically along an interface between two half-planes of linear elastic material with shear modulus varying spatially in the direction transverse to the fracture plane. The mathematical difficulty stems from the fact that elastic waves are dispersive in such a medium due to the spatial dependence of the sound speed in the bulk material. To study this

problem, they performed an asymptotic analysis based upon the assumption that the material properties through out the bulk material are given by an asymptotic series with zero order term corresponding to a homogeneous elastic material.

Since there are almost no analytical solutions to dynamic, unsteady crack problems in nonhomogeneous elastic material available in the literature against which to check the range of validity of the asymptotic solution described above, Zeigler and the P.I. applied their asymptotic approach to the simpler quasi-static problem for which one can construct analytic solutions for a few special classes of nonhomogeneous material models. Numerical results show quite reasonable agreement between the asymptotic approximation to the Stress Intensity Factor (SIF) and the true SIF for this quasi-static problem. The P.I. and his current Ph.D. student, Dinara Khalmanova, are now developing a computer code to simulate crack growth for the full transient problem based upon the asymptotic stress solution. Completion of this phase of the project is still several months away.

The P.I. has begun a collaboration with Dr. Rob Lipton (LSU) to carry out a systematic study of optimizing the grading of material properties in thin bonding layer joining two different homogeneous elastic materials in a composite laminate in order to maximize its fracture resistance. Various structural scenarios and loading modes are being considered. For example, first they are attempting to determine what grading pattern within the bond layer (whose material properties at its two interfaces are constrained to match those of the corresponding adjoined homogeneous material layers) minimizes the maximum shear stress occurring within the bonding layer under the assumption that no crack exists along either interface. The goal is to see what grading pattern minimizes the likelihood that a crack will form along either interface or within the bond layer. Several specific problems were formulated by the P.I. and Lipton while the P.I. visited him at LSU for two days in February 2003. Work on those problems is ongoing.

**Temperature Effects in Dynamic Fracture.** In collaboration with F. Costanzo, the P.I. continued consideration of temperature effects in dynamic fracture. This work is motivated by experimental and computational studies of dynamic fracture in brittle polymers. On the experimental side, it has been observed that very dramatic temperature rises can occur in the vicinity of dynamically propagating crack tips in brittle polymers. One normally associates such temperature rises with metal plasticity, however Costanzo and the P.I. conjecture that in polymers, the temperature rise is due in part to the frictional type resistance encountered in the fibril drawing out process occurring in the crack tip craze zone that always accompanies fracture in polymers.

On the computational side, it has been shown for certain lattice fracture models of brittle fracture, that dynamic steady-state crack-tip propagation is untenable below a certain threshold speed. Moreover, numerical simulations with some of these lattice fracture models seem to show a transition to an instability characterized by rapid

crack-tip speed oscillations. These features seem difficult to capture in purely mechanical continuum models since they reflect localized fine scale vibrational modes. Costanzo and the P.I. conjectured that such features might emerge from continuum models through the consideration of temperature effects. To test this conjecture, they analyzed the canonical dynamic, steady-state problem of a semi-infinite crack propagating at constant velocity through an infinite linear thermoelastic body. Since the temperature rise accompanying dynamic crack propagation is highly localized to a thin strip surrounding the crack tip, they modeled this effect by assuming that the heat generated by the moving crack tip is produced only in a crack tip cohesive zone. The cohesive zone constitutive properties were assumed to be temperature dependent, and hence become inhomogeneous as the crack propagates.

An important goal of this effort is to understand the influence of thermal softening in the cohesive zone upon crack growth evolution and the transition from stable crack growth to the unstable regime observed experimentally. Various non-linear, rate dependent, thermomechanical cohesive zone constitutive models were studied, and simulations were run for a crack propagating in the brittle polymer PMMA. One of the several interesting features observed in these simulations is that the cohesive stress forms a sharp peak near the leading edge of the cohesive as the crack speed approaches the limiting wave speed. In fact these predicted stress levels would be unsustainable in real material, suggesting that the cohesive zone would in fact fragment before these speeds were attained. This observation provides an explanation for why steady crack speeds above around 60% of the Rayleigh wave speed have not been observed experimentally, and suggests a mechanism for the transition from a smoothly accelerating tip phase to an unstable regime with oscillating crack tip speeds. Namely, attempts to drive a crack at such high speeds initiates a periodic like process of cohesive zone fragmentation and regrowth. It follows, that steady state conditions at such speeds can never be attained. Correspondingly, the simulations show that the dynamic fracture process produces a dramatic temperature rise in the cohesive zone. This work appears in the paper [[7]].

The P.I. has been collaborating with Costanzo on developing a computational model based upon a discontinuous Galerkin method for the cohesive zone fragmentation problem including the effects of temperature. Work is ongoing and the P.I. and Costanzo hope to have a working code in the near future.

**Dynamic Steady-State Crack Growth in an Anisotropic Viscoelastic Body.** The P.I. continued his study of dynamic, steady-state crack problems in a general, linear anisotropic viscoelastic body. The intended application is to composite materials whose material properties can be approximated by those of a homogeneous material exhibiting anisotropy and hysteresis under cyclic loading. While some analytical studies exist in the literature (or in pre-print form) of dynamic steady-state crack propagation in an anisotropic elastic material, the corresponding viscoelastic problem has not been addressed. This is due partly to the fact that the approach developed by Stroh that is has been the basis for nearly all existing analytical work

on the elastic problem, does not generalize to the viscoelastic case. The P.I. has developed a general solution method based upon transform and complex variable techniques that produces a full field solution to a semi-infinite crack problem under general crack face loadings leading to generalized plane strain deformations. A key step in the analysis is the construction of the Dirichlet-to-Neumann map on the bounding plane for an anisotropic viscoelastic half-space under dynamic steady-state conditions. The construction of the D-N map requires the analysis of the anisotropic viscoelastic Rayleigh function. The P.I. had intended to appeal to recent work by M. Romeo which proved the uniqueness of Rayleigh waves across the surface of an anisotropic viscoelastic half-space. Surprisingly, even the result for an isotropic viscoelastic half-space is new. Part of the problem here is that Hays and Currie gave an asymptotic argument in the mid 1980's suggesting that multiple Rayleigh waves could exist along a viscoelastic half-space which had no elastic counterpart. Were this true, the technique used by the P.I. to solve the fracture problem would break down. However, Romeo proved that the extra Rayleigh waves claimed by Hayes and Currie for a viscoelastic material are in fact spurious and don't satisfy the viscoelastic Rayleigh equation. Unfortunately, the main result of Romeo has a gap in the proof. While Romeo has given correct arguments that the work of Hays et al is incorrect, the issue of the possibility of multiple roots for the Rayleigh function remains open. Nevertheless, the P.I. has shown that the needed result is true for various special cases of anisotropy most relevant to composites. He is now working out numerical simulations based upon his solution with his graduate student Dinara Khalmanova. This work appears in the paper [[10]].

**Dynamic Unsteady Crack Propagation in a Linear Viscoelastic Body.** The P.I. along with his former student T. Leise (Rose-Hulman) generalized their method for solving dynamic unsteady crack problems in an isotropic linear elastic body to a viscoelastic body [3]. Previously they had applied their approach to studying multiple co-planar cracks [2, 1] in order to simulate the growth and coalescence of a system of (anti-plane shear) micro-cracks with a main macro-crack. Such growth and coalescence of micro-cracks is thought to be a common mechanism by which macro-cracks grow in brittle materials. They applied their approach to a single semi-infinite anti-plane shear crack in a viscoelastic body. This solution is the first dynamic, unsteady crack analysis valid for a general linear viscoelastic body and shows dramatically the critical effect viscoelasticity has upon predictions of crack length and speeds obtained for given loadings.

**Dynamic Steady Crack in a Viscoelastic Body Driven by a Rigid Wedge.** In joint work with Yuri Antipov (LSU), the P.I. has begun to analyze the dynamic steady state problem of a crack driven through a viscoelastic body by a rigid wedge. The problem is relevant to an important experimental technique for generating dynamically running cracks as well as to projectile impact and fracture of brittle polymers. One of the key questions to be addressed is determining the fracture profile and energetics as the wedge speed approaches the critical Rayleigh wave speed. This work, which is relevant to a central question in dynamic fracture concerning the limiting

speed of cracks, is ongoing.

**Finite Elastic Deformations.** In addition to considering material models for brittle polymers, the P.I. has been investigating the large deformation behavior of rubbery materials. The setting has been finite elasticity and two themes have been pursued: the convexity or strong ellipticity properties of anisotropic finite elastic constitutive models; the stability of various classes of solutions to finite elastic boundary value problems for a wedge shaped body. In the former problem area, the P.I. and his former postdoc Patrick Wilber showed that the classical anisotropic constitutive model due to Y. C. Fung fails to be strongly elliptic. They also discuss various approaches to modeling anisotropic finite elastic behavior through which strong ellipticity or convexity properties can be guaranteed. These results, which are contained in the two papers [4, 5] have important implications for modeling material behavior and for doing numerical computations. This work is now being generalized. In the latter problem area, the P.I. demonstrates that several classes of plane strain, large deformation solutions for the infinite elastic wedge problem are unstable to small scale vibrational perturbations [8]. He also showed that this property is a nonlinear elastic effect that does not occur for the linear elastic approximation. Subsequently, he and P. Wilber studied the issue of solutions to the wedge problem with uniformly bounded stresses. A number of previous researchers had shown that a certain class of homogeneous deformations lead to bounded stress, and they exhibited a few classes of nonhomogeneous deformations exhibiting stress singularities at the wedge apex. The P.I. and Wilber showed [6] that even among a very wide class of fully three dimensional nonhomogeneous deformations, none exist with bounded stresses at the apex; only homogeneous deformations can lead to bounded stresses. These results have implications for predicting the behavior of rubbery shock absorbers.

**Polymer Flows.** The P.I. in collaboration with L-I. Palade and A. Farina studied the stability properties of a new class of constitutive models for extensional flows of nonlinear viscoelastic fluids [9]. The issue addressed was how to incorporate the effects of widely differing polymer chain lengths in the polymer melt. The models have implication for polymer processing.

### Personnel.

The P.I. and his students Dinara Khalmanova (degree expected May 2004) and David Zeigler (degree conferred August 2001) were the only personnel receiving partial support from the grant funds.

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### **Interactions.**

The P.I. spoke on part of this research in an invited talk to the Symposium on Dynamic Fracture at the Annual meeting of the Society of Engineering Science in November 1999.

The P.I. spoke on part of this research in an invited talk to the Prager Symposium at the Annual meeting of the Society of Engineering Science in October 2000.

In December 2001, the P.I. gave a seminar at the University of Florida's Graduate Engineering Research Center at Eglin AFB to GERC and AFRL personnel. He returned to the GERC in April 2002 with his collaborator Dr. Francesco to give another seminar at the GERC. In December the P.I. had discussions with Dr. Davy Belk of the AFRL and some of his colleagues about their interests in fracture and more general damage models. In April, the P.I. met with Dr. Molly Hughés (Damage Mechanisms Branch of the Ordnance Division within the Munitions Directorate at the AFRL/ Eglin AFB) to discuss their interests in functionally graded materials.

In April 2000, the P.I. met with Dr. N. Morley, Dr. W. P. Latham and Dr. J. K. Chen of the Directed Energy Laser Effects (DELE) group at the Kirtland AFB Air Force Research Laboratory to explore establishing a collaborative research interaction. A second meeting was held in June 2001 that included the P.I.'s collaborator from Penn State University, Dr. F. Costanzo and Dr. Arje Nachman of the AFOSR. One of the DELE's tasks is to model and simulate laser induced dynamic fracture of a pressurized vessel. It was acknowledged by the DELE scientists that the classical fracture mechanics models currently employed to simulate dynamic fracture probably are incapable of providing realistic predictions, and that the models being studied by the P.I. and Dr. Costanzo show promise of producing more realistic simulation capabilities.

In 2002, the P.I. gave invited talks on different aspects of this research at the following three symposia: Symposium on Dynamic Fracture held at the 39th Annual Meeting of the Society of Engineering Science, State College, PA, October 2002; Symposium in Honor of R. A. Schapery held at the 14th U.S. National Congress of Applied Mechanics, Blacksburg, VA, July 2002; Symposium in Honor of W. G. Knauss held at the 14th U.S. National Congress of Applied Mechanics, Blacksburg, VA, July 2002.

One of the reviewers of the P.I.'s 2002 AFOSR renewal proposal, Dr. Ajit Roy of the AFRL at Wright-Patterson AFB, suggested that the P.I. consider applications of his work on fracture in nonhomogeneous materials to carbon foam materials. In response to this suggestion, the P.I. contacted Dr. Ozden Ochoa, a faculty member in the Mechanical Engineering Dept. at Texas A&M University currently on leave as a senior scientist at the same AFRL division as Dr. Roy, who is working on testing and modeling carbon foam materials. The P.I. met with Dr. Ochoa and separately with her student to discuss problems associated with modeling these complex materials. Dr. Ochoa and her student have done some preliminary experiments on the fracture of carbon foam composites, but have not yet attempted to model the fracture process. The P.I. intends to continue these discussions which hopefully will lead to an active collaboration to model fracture in carbon foam composites.